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OVER VIEW OF LAPLACE TRANSFORMS AND ITS APPLICATIONS

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ABSTRACT

Laplace transform is a very powerful mathematical tool applied in various areas of engineering and science. With the increasing complexity of engineering problems, Laplace transforms help in solving complex problems with a very simple approach just like the applications of transfer functions to solve ordinary differential equations. The method of the Laplace transform has found an increasing number of applications in the fields of physics and technology. In this paper will discuss the Properties and applications of Laplace transforms.

KEYWORDS: Integral Transform, Laplace Transform, Domain, Electrical Circuits.

HISTORY:
The Laplace Transform was first used and named after by Pierre Simon Laplace. Pierre Simon Laplace was a French Mathematician and Astronomer, who had a lot of influence in the development of several theories in mathematics, statistics, physics, and astronomy. He contributed greatly to physical mechanics, by converting the old geometrical analysis to one based on calculus, which opened up application of his formulas to a wider range of problems.

INTRODUCTION:
Laplace transform is an integral transform method which is particularly useful in solving linear ordinary differential equations. It finds very wide applications in various areas of physics, electrical engineering, control engineering, optics, mathematics and signal processing. The Laplace transform can be interpreted as a transformation from the time domain where inputs and outputs are functions of time to the frequency domain where inputs and outputs are functions of complex angular frequency. In order for any function of time f(t) to be Laplace transformable, it must satisfy the following Dirichlet conditions

- The function f(t) has finite number of maxima and minima.
- There must be finite number of discontinuities in the signal f(t),in the given interval of time.
- It must be absolutely integrable in the given interval of time. i.e.

$$\int_{-\infty}^{\infty} |f(t)|dt < \infty.$$

Let f(t) be a real-valued function of the real variable t defined on the positive portion of the real axis, for $t \geq 0$. Then the Laplace transform of f(t) is denoted by L {f(t)} and is defined as

$$F(S) = L\{f(t)\} = \int_0^{\infty} e^{-st} f(t)dt$$

The integral which defined a Laplace transform is an improper integral. An improper integral may converge or diverge, depending on the integrand. When the improper integral is convergent then we say that the function f(t) possesses a Laplace transform. Whenever the limit exists (as a finite number) the integral is said to converge. If the limit does not exist, the integral is said to diverge and there is no Laplace transform defined for f(t).

In a layman's term, Laplace transform is used to "transform" a variable in a function into a parameter. So, after the transformation that variable is no longer a variable anymore, but should be treated as a "parameter", i.e a "constant under a specific condition". This "specific condition" for Laplace transform stipulates that, Laplace transform can only be used to transform variables that cover a range from zero (0) to infinity (∞) that is $0 < t < \infty$. Any variable that does not vary within this range cannot be transformed using Laplace transform. Laplace transform is a valuable "tool" in solving differential equations for example: electronic circuit equations and in "feedback control" systems, in stability and control of aircraft systems. Because time variable t is most common variable that varies from 0 to ∞, functions with variable t are commonly transformed by Laplace transform. Basic Definitions and Results Laplace transform is yet another operational tool for solving constant coefficients linear differential equations. The process of solution consists of three main steps:

- The given "hard" problem is transformed into a "simple" equation.
- This simple equation is solved by purely algebraic manipulations.
- The solution of the simple equation is transformed back to obtain the solution of the given problem.

In this way the Laplace transformation reduces the problem of solving a differential equation to an algebraic problem. The third step is made easier by tables, whose role is similar to that of integral tables in integration. Ordinary and partial differential equations describe the way certain quantities vary with time, such as the current in an electrical circuit, the oscillations of a vibrating membrane or the flow of heat through an insulated conductor. These equations are generally coupled with initial conditions that describe the state of the system at time t. A very powerful technique for solving these problems is that of the Laplace transform, which literally transforms the original differential equation into an elementary algebraic expression. This latter can then simply be transformed once again, into the solution of the original problem. This technique is known as the "Laplace transform method."

Some Important Properties of Laplace Transforms
Property-1(Constant Multiple):
If 'a' is a constant and f(t) is a function of 't' then

$$L\{af(t)\}=aL\{f(t)\}$$

Property-2(Linearity Property):
If 'a' and 'b' are constants while f(t) and g(t) are functions of 't' then

$$L\{af(t)+bg(t)\}=aL\{f(t)\}+bL\{g(t)\}$$

Property-3(Change of scale Property):
If $L\{f(t)\}=F(s)$ then

$$L\{f(at)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$$

Property-4(Shifting Property):
If $L\{f(t)\}=F(s)$ then

$$L\{e^{at} f(t)\}=F(s-a)$$

Property-5:
If $L\{f(t)\}=F(s)$ then

$$L\{tf(t)\} = -F'(s) = -\frac{d}{ds}F(s)$$

Property-6:
If $L\{f(t)\}=F(s)$ then

$$L\{f'(t)\}=SF(s)-f(0)$$

Property-7:
If L{f(t)}=F(s) and $g(t) = \frac{f(t)}{t}$ then

$$L\{g(t)\} = G(s) = \int_s^{\infty} F(p)dp$$

Property-8:
If $L\{f(t)\}=F(s)$ then

$$L\left[\int_0^t f(t)dt\right]=\frac{F(s)}{s}$$

APPLICATIONS OF LAPLACE TRANSFORMS
Laplace transform has several applications in almost all Engineering disciplines.

- **System Modeling**

Laplace transform is used to simplify calculations in system modeling, where large differential equations are used.

- **Analysis of Electrical Circuits**

In electrical circuits, a Laplace transform is used for the analysis of linear time-invariant systems.

- **Analysis of Electronic Circuits**

Laplace transform is widely used by Electronics engineers to quickly solve differential equations occurring in the analysis of electronic circuits.

- **Digital Signal Processing**

One cannot imagine solving DSP (Digital Signal processing) problems without employing Laplace transform.

- **Nuclear Physics**

In order to get the true form of radioactive decay, a Laplace transform is used. It makes studying analytic part of Nuclear Physics possible.

- **Process Controls**

Laplace transforms are critical for process controls. It helps analyze the variables, which when altered, produces desired manipulations in the result. For example, while study heat experiments, Laplace transform is used to find out to what extent the given input can be altered by changing temperature, hence one can alter temperature to get desired output for a while. This is an efficient and easier way to control processes that are guided by differential equations.

- **Application in probability**

It is also called Moment. It is used to mathematically define variables in probability like variance (second degree moment) etc...

- **Application in Physics**

A very simple application of Laplace transform in the area of physics could be to find out the harmonic vibration of a beam which is supported at its two ends

- **Application in Electric Circuit Theory**

The Laplace transform can be applied to solve the switching transient phenomenon in the series or parallel RL, RC or RLC circuits

- **Application in Power Systems Load Frequency control**

Power systems are comprised of generation, transmission and distribution systems. A generating system consists of a turbo generator set in which a turbine drives the electrical generator and the generator serves the loads through transmission and distribution lines.

- Laplace transformation techniques made rigorous earlier adhoc operator methods, in which the differential with respect to time is replaced by an operator D, with 1/D being integration. The operator D is then treated as if it is an algebraic quantity.

- The operator technique was fully developed by the physicist Oliver Heaviside in 1893, in connection with his work in telegraphy. Guided greatly by intuition and his wealth of knowledge on the physics behind his circuit studies, Heaviside developed the operational calculus now ascribed to his name.

- A rigorous mathematical justification of Heaviside's operational methods came only after the work of Bromwich that related operational calculus with Laplace transformation methods.

- In computer science it is hardly used, except maybe in data mining/machine learning.

- The primary application is that it lets you solve several differential equations by transforming them to simpler differential equations. Since all sorts of problems in science and engineering can be reduced to differential equations, this makes it an extremely applicable technique. Laplace transform converts complex ordinary differential equations (ODEs) into differential equations that have polynomials in it. Solving an equation with polynomials in it is easier; this is why we use it. It reduces the complexity of the problem and time taken to solve it.

- Laplace transform will convert a function in some domain into a function in another domain, without changing the value of the function.

- We use Laplace transform on a derivative to convert it into a multiple of the domain variable. Thus with Laplace transform nth degree differential equation can be transformed into an nth degree polynomial.

- One can easily solve the polynomial to get the result and then change it into a differential equation using inverse Laplace transform.

- A simple Laplace Transform is conducted while sending signals over any two-way communication medium (FM/AM stereo, 2-way radio sets, cellular phones).

- When information is sent over medium such as cellular phones, they are first converted into time-varying wave, and then it is super-imposed on the medium.

- In this way, the information propagates. Now, at the receiving end, to decipher the information being sent, medium wave's time functions are converted to frequency functions.

- Laplace transform is really just a shortcut for complex calculations. It may seem troublesome, but it bypasses some of the most difficult mathematics.

CONCLUSION:
The paper presented the application of Laplace transform in different areas of physics and electrical power engineering. Besides these, Laplace transform is a very effective mathematical tool to simplify very complex problems in the area of stability and control. With the ease of application of Laplace transforms in myriad of scientific applications, many research soft wares have made it possible to simulate the Laplace transformable equations directly which has made a good advancement in the research field.

The Fourier transform and Laplace transform are related. The Fourier transform resolves functions or signal into its mode of vibration whereas the Laplace transform resolves a function into its moments. Both are used for designing electrical circuits, solving differential and integral equations.

REFERENCES:
[1] A. D. Poularikas, The Transforms and Applications Handbook (McGraw Hill, 2000), 2nd ed.

[2]. G. B. Folland,.: Fourier Analysis and its Application, Pacific Grove, CA Brooks / Cole. 1992

[3]. M .Abramovitz, and Stegun Laplace Transforms, New York Dover. 1972 [4]. K.A. Stroud "Advanced Engineering Mathematics" Mac Williams 2003, ISBN 1-4039-0312-3.

[5]. H.k. Dass "Advanced Engineering mathematics" S.chand and company ltd New Delhi, 2009.